

Expanding and Factoring are opposite operations.

Examples

Expand $(x + 5)(x + 2)$

$$= x^2 + 2x + 5x + 10$$

$$= x^2 + 7x + 10$$

Notice!

$$5 \times 2 = 10$$

$$5 + 2 = 7$$

Factor $x^2 + 7x + 10$

?

Expand $(x + 4)(x + 1)$

Notice!

$$4 \times 1 = 4$$

$$4 + 1 = 5$$

Expand $(x + 3)(x + 4)$

Notice!

$$3 \times 4 = 12$$

$$3 + 4 = 7$$

When Factoring Trinomials, we need to find...

$$x^2 + bx + c$$

Two numbers that add together to equal b.

Two numbers that multiply together to equal c.

$$\begin{array}{rcl} \triangle & \times & \square \\ \hline \triangle & + & \square \end{array} = c$$

$$\begin{array}{rcl} \triangle & + & \square \\ \hline \triangle & + & \square \end{array} = b$$

So that we get: $(x + \triangle)(x + \square)$

Example 1: Factor each trinomial.

a) $x^2 + 7x + 12$

$$\begin{array}{rcl} 3 & \times & 4 \\ \hline 3 & + & 4 \end{array} = 12$$

$$\begin{array}{rcl} 3 & + & 4 \\ \hline 3 & + & 4 \end{array} = 7$$

$$= (x + 3)(x + 4)$$

b) $x^2 + 6x + 8$

$$\begin{array}{rcl} 2 & \times & 4 \\ \hline 2 & + & 4 \end{array} = 8$$

$$\begin{array}{rcl} 2 & + & 4 \\ \hline 2 & + & 4 \end{array} = 6$$

$$= (x + 2)(x + 4)$$

c) $x^2 + 3x - 4$

$$\begin{array}{rcl} -1 & \times & 4 \\ \hline -1 & + & 4 \end{array} = -4$$

$$\begin{array}{rcl} -1 & + & 4 \\ \hline -1 & + & 4 \end{array} = 3$$

$$= (x - 1)(x + 4)$$

d) $x^2 - 3x - 18$

$$\begin{array}{rcl} -6 & \times & 3 \\ \hline -6 & + & 3 \end{array} = -18$$

$$\begin{array}{rcl} -6 & + & 3 \\ \hline -6 & + & 3 \end{array} = -3$$

$$= (x - 6)(x + 3)$$

$$\begin{array}{rcl} -9 & \times & 2 \\ -3 & \times & 6 \\ -1 & \times & 18 \\ 1 & \times & -18 \\ -6 & \times & 3 \\ -2 & \times & 9 \end{array}$$

Example 2: Find an expression for the rectangle's area by factoring.

$$A = L \times w$$

$$A = x^2 + 3x - 28$$

$$\begin{array}{r} -28 \\ 2x-14 \\ \underline{-4 \times 7} \\ -2 \times 14 \end{array}$$

$$A = (x-4)(x+7)$$

$$L = x-4$$

$$w = x+7$$

$$\begin{array}{r} -4 \times 7 = -28 \\ -4 + 7 = 3 \\ -28 \times 1 \\ -1 \times 28 \end{array}$$

Example 3: Factor the following. (**Hint:** You will need to common factor first and then trinomial factor second.)

$$\begin{aligned} \text{a) } & 5x^2 + 35x + 30 \\ & = 5 \left(\frac{5x^2 + 35x + 30}{5} \right) \end{aligned}$$

$$= 5(x^2 + 7x + 6)$$

$$\frac{6}{6 \times 1} \times \frac{1}{3 \times 2} = 6$$

$$\frac{6}{6} + \frac{1}{1} = 7$$

$$= 5(x+6)(x+1)$$

$$\begin{aligned} \text{b) } & -2x^2 + 4x - 2 \\ & = -2 \left(\frac{-2x^2 + 4x - 2}{-2} \right) \end{aligned}$$

$$= -2(x^2 - 2x + 1)$$

$$\frac{-1}{1 \times 1} \times \frac{-1}{1 \times 1} = 1$$

$$\frac{-1}{-1 \times -1} + \frac{-1}{-1 \times -1} = -2$$

$$= -2(x-1)(x-1)$$