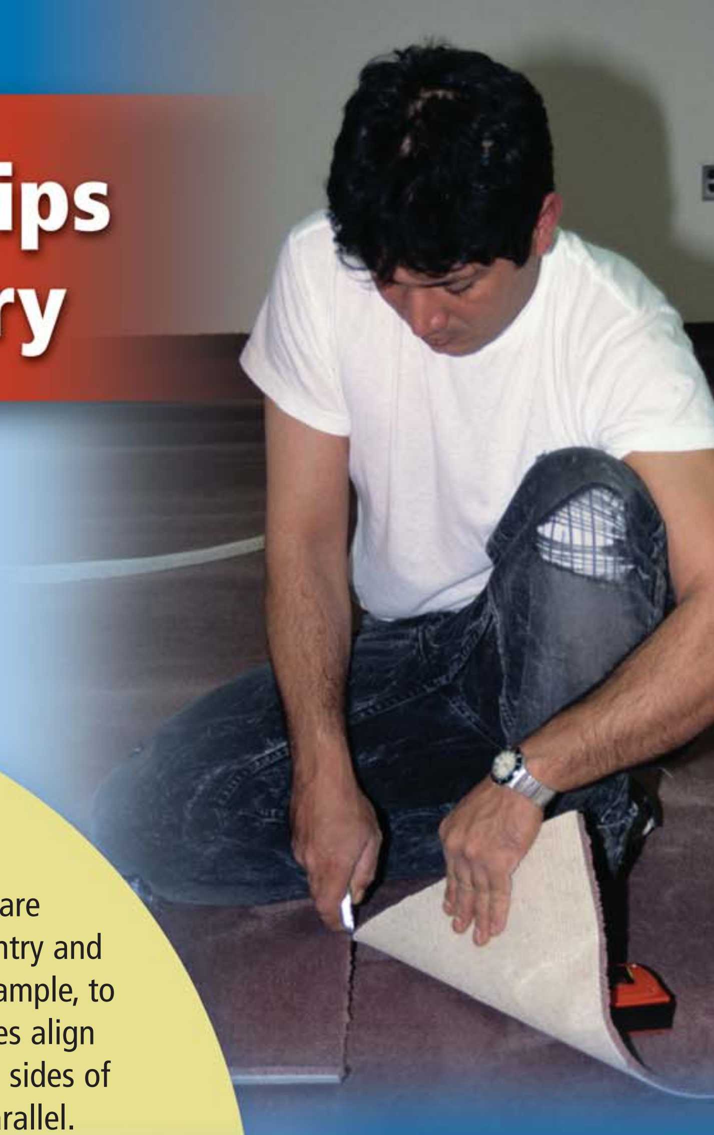
3 Relationships in Geometry



What You'll Learn

The properties and relationships among angles in polygons and angles involving parallel lines

And Why

These relationships are often used in carpentry and construction; for example, to ensure that floor tiles align correctly or that the sides of a door frame are parallel.

Key Words

- interior angle
- exterior angle
- transversal
- corresponding angles
- alternate angles

Project Link

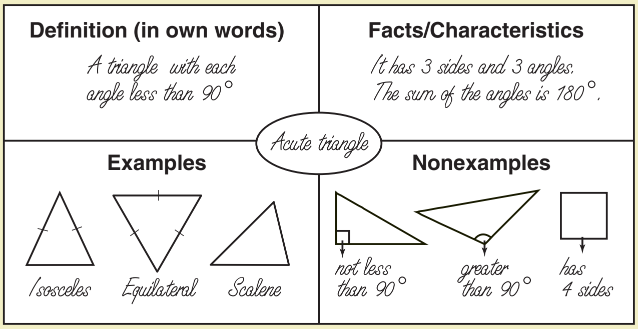
- Designing with Shape, Colour, and Motion

Using a Frayer Model

Every math topic has words and concepts for you to learn.

A Frayer model is a tool that helps you visualize new words and concepts.
It makes connections to what you already know.

Here is an example of a Frayer model.



See if you can figure out which word goes in the centre of this Frayer model.

Definition (in own words)	Facts/Characteristics
A figure with 4 sides and 4 angles	The sides are straight. The figure is closed.
?	
Examples	**Nonexamples**
Square Rectangle Parallelogram Trapezoid Kite	Triangle Pentagon Circle Cube

Make your own Frayer model for each of these concepts:
- Isosceles triangle
- Pythagorean Theorem

As you work through this chapter, look for ways to use a Frayer model.
Scan each section for new words and new concepts.
Try to make at least 3 Frayer models in this chapter.
Here are some suggestions:
- Exterior angle
- Parallel lines
- Quadrilateral

3.1 Angles in Triangles

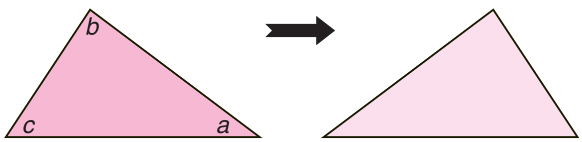
Investigate The Sum of the Interior Angles in a Triangle

Draw *large* triangles.

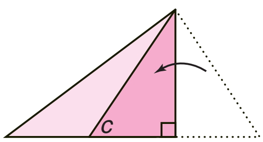
Work in a group of 3.
You will need scissors.

➢ Use a ruler. Each of you draws one of these triangles: acute triangle, obtuse triangle, and right triangle. Cut out the triangles.

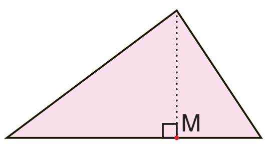
➢ Place each triangle so its longest side is at the bottom and its shortest side is to your left. Label the angles as shown. Turn over the triangle so the labels are face down.



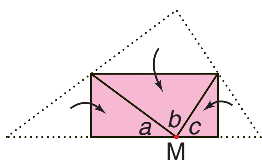
➢ Make a fold as shown. The crease is at right angles to the bottom side.



➢ Unfold the triangle and lay it flat. Mark the point M where the crease meets the bottom side.



➢ Fold each vertex of the triangle to meet at point M.



➢ How do the three angles of the triangle appear to be related?

Reflect

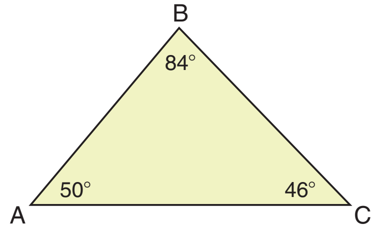
➢ Compare your results with those of your group.

➢ Did everyone get the same relationship? Explain.

Connect the Ideas

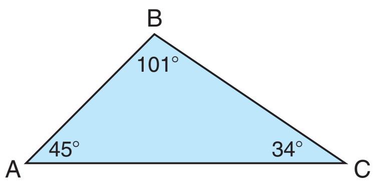
The 3 angles in a triangle are its **interior angles**.

Zahara constructs a triangle using *The Geometer's Sketchpad*. She measures the angles in the triangle and calculates their sum.

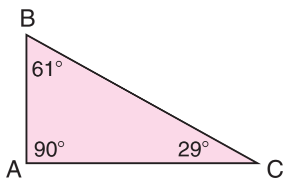


$\angle A + \angle B + \angle C = 180°$
Acute triangle

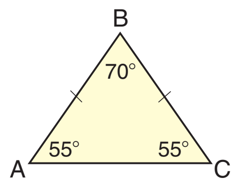
Zahara drags the vertices to create different types of triangles. The angle measures change, but their sum does not.



$\angle A + \angle B + \angle C = 180°$
Obtuse triangle

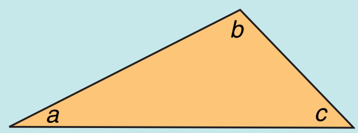


$\angle A + \angle B + \angle C = 180°$
Right triangle



$\angle A + \angle B + \angle C = 180°$
Isosceles triangle

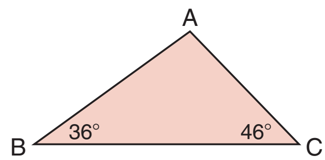
The sum of the three interior angles of a triangle is 180°.
That is, $a + b + c = 180°$



We can use this relationship to determine the third angle in a triangle when we know the other two angles.

In $\triangle ABC$, $\angle A + \angle B + \angle C = 180°$

$\angle B + \angle C = 36° + 46°$
$= 82°$

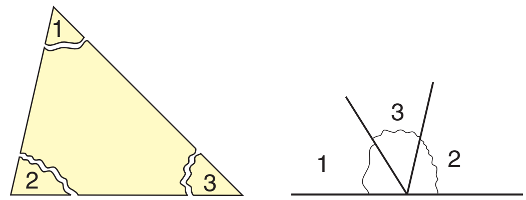


To determine $\angle A$, think:
What do we add to 82° to get 180°?
Subtract to find out.
$180° - 82° = 98°$

So, $\angle A = 98°$

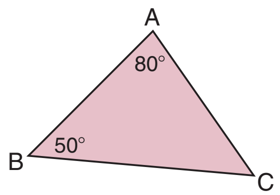
Practice

1. Use a ruler to draw a large triangle.
Cut out the triangle and tear off its corners.
Put the corners together as shown.
Which relationship does this illustrate? Explain.

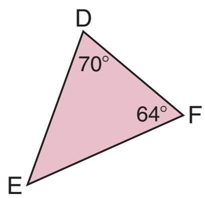


2. Determine each unknown angle.

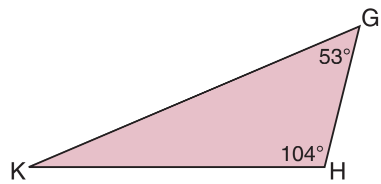
a)



b)



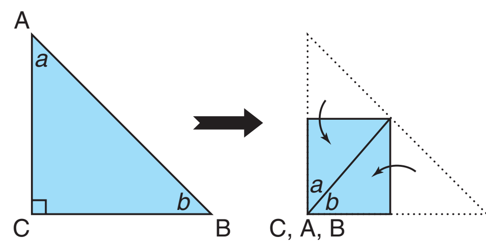
c)



3. For each set of angles, can a triangle be drawn?
Justify your answers.
a) 41°, 72°, 67° b) 40°, 60°, 100° c) 100°, 45°, 30°

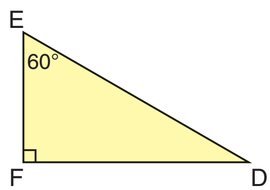
4. Can a triangle have two 90° angles? Explain.

5. **Assessment Focus**
a) Draw then cut out a right triangle. Fold the triangle as shown. What does this tell you about the acute angles of a right triangle?
b) Use what you know about the sum of the angles in a triangle to explain your result in part a.



c) Determine each unknown angle. Explain how you did this.

i)



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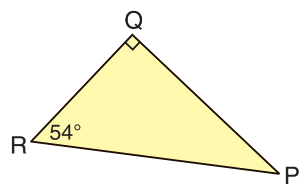

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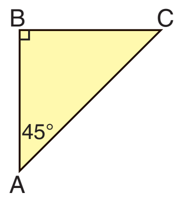
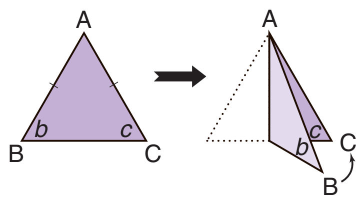
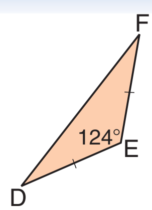


6. Draw then cut out an isosceles triangle.
Fold the triangle as shown.
What does this tell you about the angles opposite the equal sides of an isosceles triangle?



Example

a) Write a relationship for the measures of the angles in isosceles △DEF.
b) Use the relationship to determine the measures of ∠D and ∠F.



Solution a) The sum of the angles in a triangle is 180°.
So, $\angle D + \angle F + 124° = 180°$

b) $\angle D + \angle F = 180° - 124°$
$= 56°$

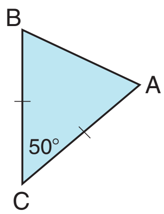
In an isosceles triangle, the angles opposite the equal sides are equal.
DE = FE, so $\angle D = \angle F$
There are 2 equal angles with a sum of 56°.
So, each angle is: $\frac{56°}{2} = 28°$

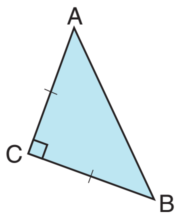
$\angle D = 28°$ and $\angle F = 28°$

> Think: What do we add to 124° to get 180°? Subtract to find out.

> Recall that $\frac{56°}{2}$ means 56° ÷ 2.

7. In each isosceles △ABC below:
- **i)** Write a relationship for the measures of the angles.
- **ii)** Use the relationship to determine the measures of ∠A and ∠B.

a)


b)


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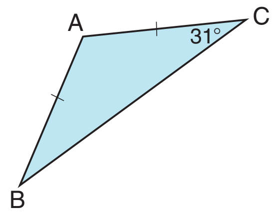

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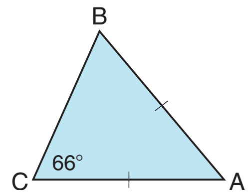
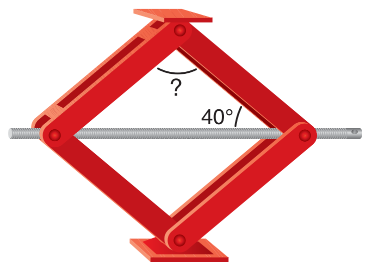
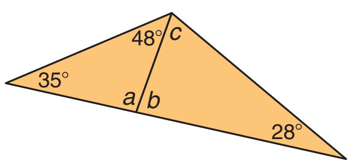


8. Most cars have a scissors jack. It has all 4 sides equal.
- **a)** The angle shown is 40°. Determine the angle at the top of the jack.
- **b)** Suppose the angle shown increases to 42°. What happens to the angle at the top of the jack? Explain how you know.



9. An equilateral triangle has 3 equal sides and 3 equal angles. Use this information to determine the measure of each angle. Show your work.

10. **Take It Further** Determine the angle measure indicated by each letter.

a)


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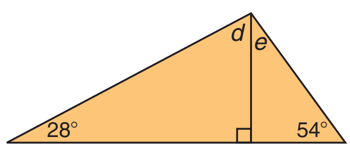

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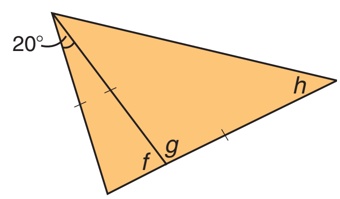


In Your Own Words

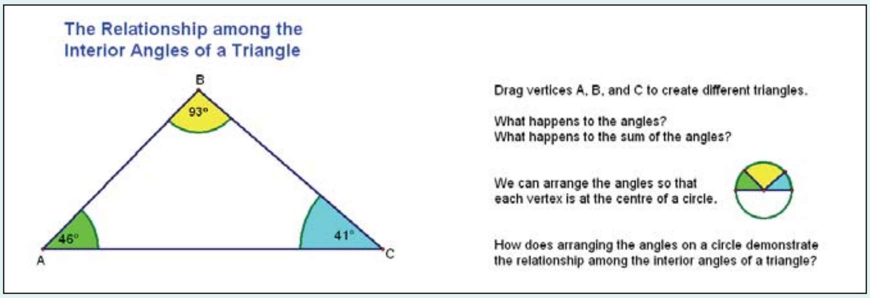
Suppose you know the measure of one angle in a triangle. What else would you need to know about the triangle to determine the measures of all its angles? Explain.

Using *The Geometer's Sketchpad* to Investigate Angles Related to Triangles

Technology

Part 1: The Relationship among the Interior Angles

1. Open the file *TriangleAngles.gsp*. Click the tab for page 1.



2. Drag vertices A, B, and C to create 5 different triangles.

a) For each triangle, copy this table and record the measures of ∠A, ∠B, and ∠C.

∠A	∠B	∠C	∠A + ∠B + ∠C

b) Calculate the sum of each set of angles.

c) How do the interior angles of a triangle appear to be related?

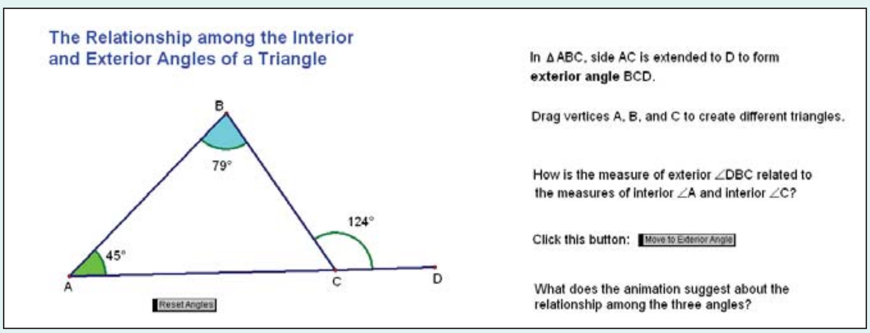
3. We can arrange ∠A, ∠B, and ∠C so the vertex of each angle is at the centre of a circle.

a) Drag vertices A, B, and C to create different triangles. Observe how the angles change on the circle.

b) How does arranging the angles on a circle support your answer to question 2c?

Part 2: The Relationship among the Interior and Exterior Angles

1. Use the file *TriangleAngles.gsp*. Click the tab for page 2.



2. In △ABC, side AC is extended to D to form exterior ∠BCD.

a) Drag vertices A, B, and C to create 5 different triangles. For each triangle, copy this table and record the measures of ∠A, ∠B, and ∠BCD.

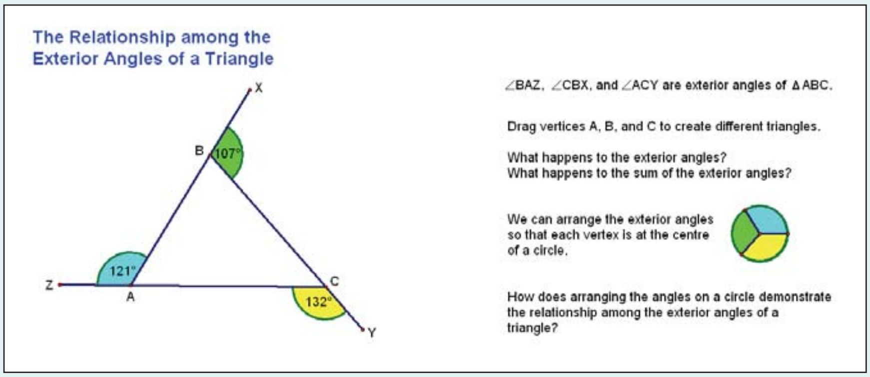
∠A	∠B	∠BCD

b) How do the three angles appear to be related? Explain why.

c) Click the button: *Move to Exterior Angle*
How does this animation support your answer to part b?

Part 3: The Relationship among the Exterior Angles

1. Use the file *TriangleAngles.gsp*. Click the tab for page 3.



2. Drag vertices A, B, and C to create 5 different triangles.

a) For each triangle, copy this table and record the measures of the exterior angles.

∠BAZ	∠CBX	∠ACY	∠BAZ + ∠CBX + ∠ACY

b) Calculate the sum of each set of angles.

c) How do the exterior angles of a triangle appear to be related? Explain why.

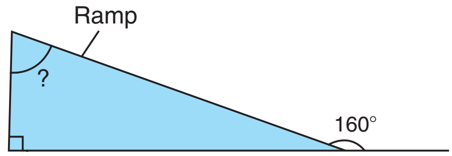
3. We can arrange ∠BAZ, ∠CBX, and ∠ACY so each vertex is at the centre of a circle.

a) Drag vertices A, B, and C to create different triangles. Observe how the angles change on the circle.

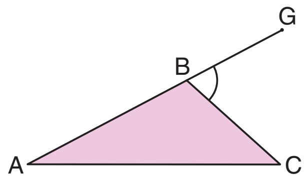
b) How does arranging the angles on a circle support your answer to question 2c?

3.2 Exterior Angles of a Triangle

Enrico is a carpenter. He is building a ramp. Enrico knows that the greater angle between the ramp and the floor is 160°. How can Enrico calculate the angle the vertical support makes with the ramp?



An exterior angle is formed outside a triangle when one side is extended.
Angle GBC is an exterior angle of $\triangle$ABC.



Investigate — Relating Exterior and Interior Angles of a Triangle

Work in a group of 3.

➢ Each of you draws one of these triangles:
  – acute triangle
  – right triangle
  – obtuse triangle

➢ Extend one side to form an exterior angle.
Measure and record the exterior angle and the interior angles.
Look for relationships between the exterior angle and one or more of the interior angles.

Reflect

Compare your results with those of your group.

➢ How is the exterior angle related to the interior angle beside it?

➢ How is the exterior angle related to the other two interior angles?

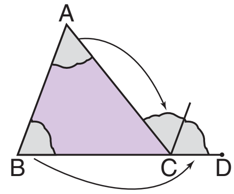
➢ Do your results depend on the type of triangle? Explain.

➢ Explain your results using facts you already know about angles and their sums.

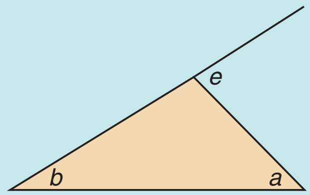
Connect the Ideas

Exterior and interior angles

Jamie cuts out a paper triangle.
He tears off the corners at A and B and finds that they fit exterior ∠ACD.
That is, ∠ACD = ∠A + ∠B



Each exterior angle of a triangle is equal to the sum of the two opposite interior angles.
That is, $e = a + b$



We can use this relationship to calculate the measure of ∠A.

∠ACD = ∠A + ∠B

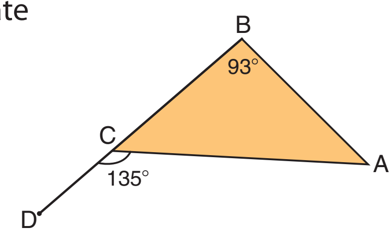
So, 135° = ∠A + 93°

Think: What do we add to 93° to get 135°?

Subtract to find out.

135° − 93° = 42°

∠A = 42°



Sum of the exterior angles

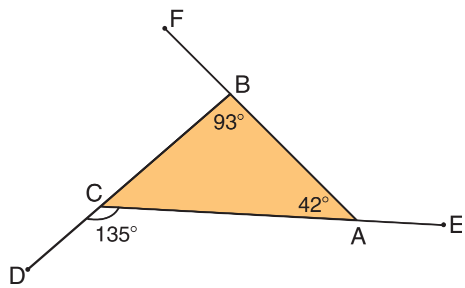
Extend the sides of △ABC to form the other exterior angles.

∠BAE = 180° − 42°
= 138°

∠FBC = 180° − 93°
= 87°

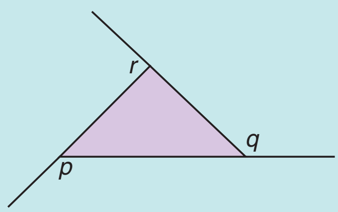
The sum of the exterior angles is:

135° + 138° + 87° = 360°



This relationship is true for any triangle.

The sum of the 3 exterior angles of a triangle is 360°.
That is, $p + q + r = 360°$



Practice

1. a) For each exterior angle, name the opposite interior angles.
b) Write a relationship between the exterior angle and the opposite interior angles.

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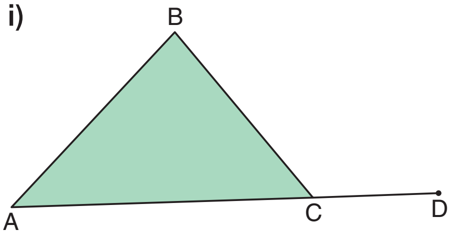

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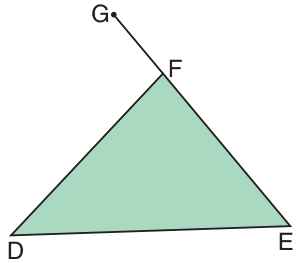

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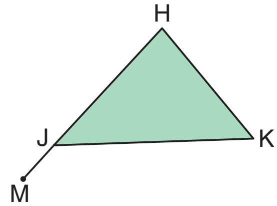

2. Determine each unknown exterior angle.
Which relationship are you using each time?

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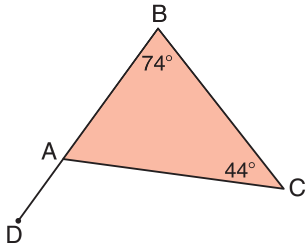

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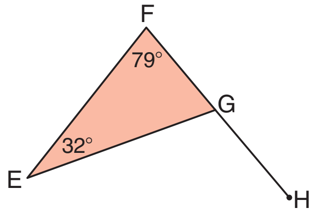

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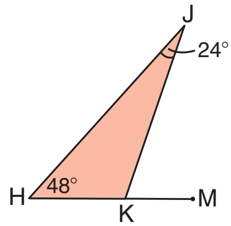

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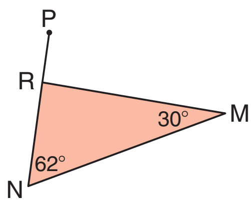

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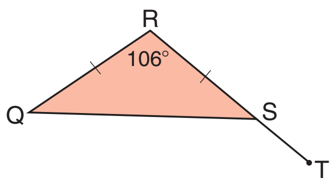

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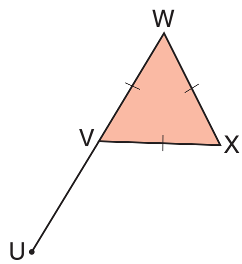
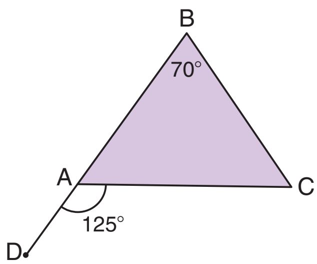
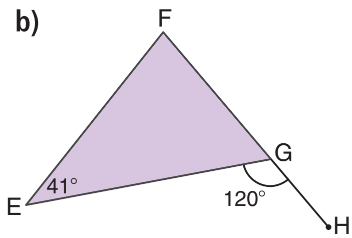
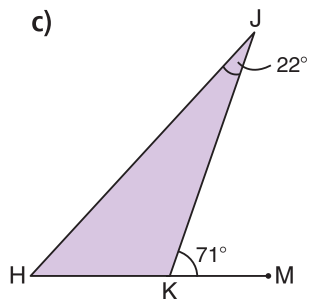


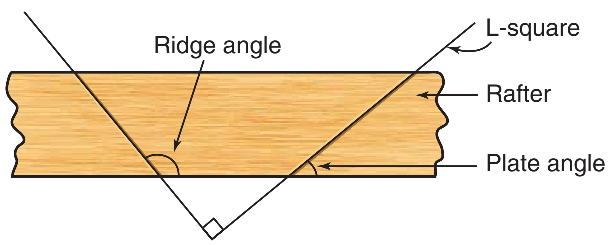
3. Determine the unknown interior angles.
Justify your answers.

a)


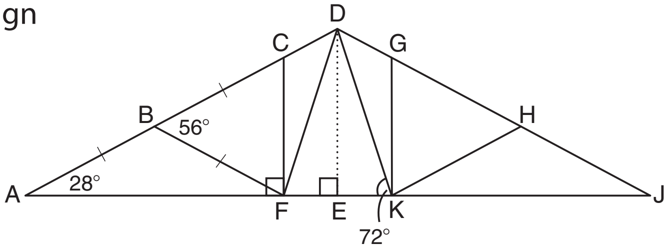
b)


c)


4. A carpenter uses an L-square to measure the timber for a rafter.
The ridge angle is 130°.
What is the plate angle?
How do you know?



5. **Assessment Focus** Grace's tech design class builds a model of a roof truss. The broken line is a line of symmetry. Determine the measure of each angle in the truss. Explain how you know.



6. Draw a triangle with an exterior angle of 160° and another exterior angle of 72°.

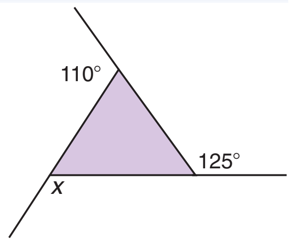
a) Measure the third exterior angle.

b) How are the three exterior angles related? Explain.

c) Compare your triangle and your results with your classmates. What do you notice?

When we know two exterior angles, we can determine the third exterior angle.

Example Determine the angle measure indicated by x.



Solution The sum of the 3 exterior angles of a triangle is 360°.

So, $x + 110° + 125° = 360°$

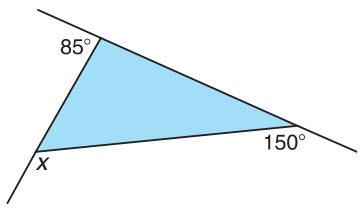
$x + 235° = 360°$

$x = 360° - 235°$

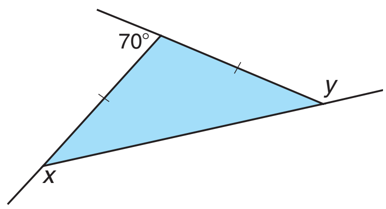
$= 125°$

7. Determine the angle measure indicated by each letter. Justify your answers.

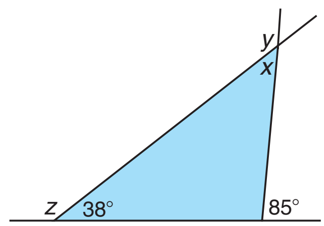
a)



b)



c)



8. **Take It Further** Sketch a triangle with two exterior angles of 150° and 130°. Describe all the different ways you can determine the measures of the third exterior angle and the three interior angles.

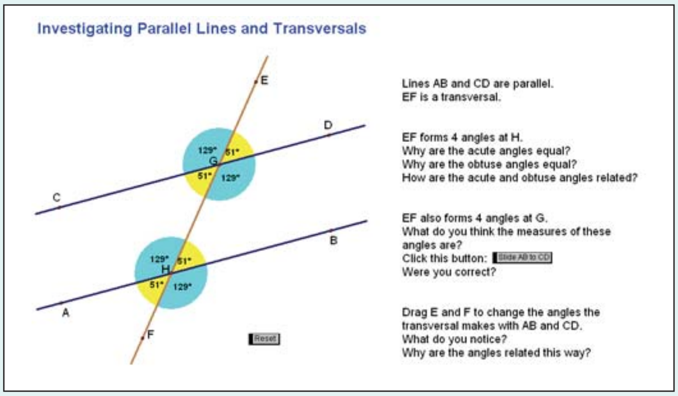
In Your Own Words

What do you know about the exterior angles of a triangle? Use diagrams to explain.

Technology

Using *The Geometer's Sketchpad* to Investigate Angles Involving Parallel Lines

1. Open the file *Parallel.gsp*. Click the tab for page 1.



2. Transversal EF intersects AB to form 4 angles at H.

a) Why are the acute angles equal?

b) Why are the obtuse angles equal?

c) How are the acute and obtuse angles related? Explain.

3. CD is parallel to AB. EF intersects CD to form 4 angles at G.

a) How do you think the angles at G are related to the angles at H? Explain.

b) Click the button: *Slide AB to CD*.

c) Were you correct in part a? Explain.

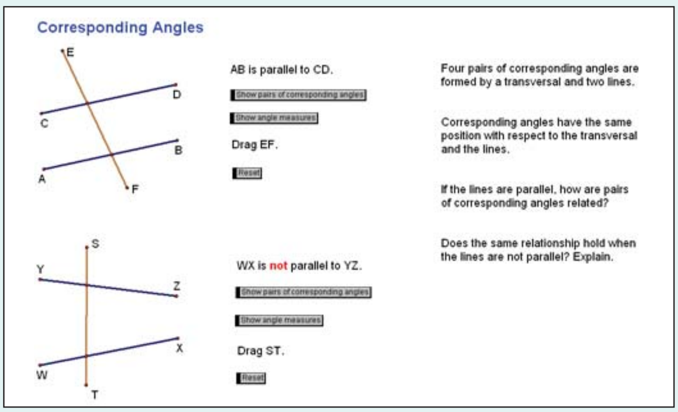
4. Drag points E and F to change the angles that EF makes with AB and CD.

a) What do you notice?

b) Why are the angles related this way?

Corresponding angles

5. Click the tab for page 2.



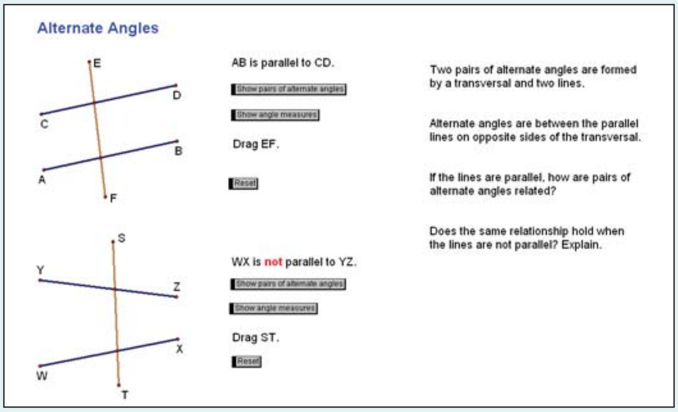
6. Four pairs of corresponding angles are formed by a transversal and two lines.

a) If the lines are parallel, how are pairs of corresponding angles related?

b) Does the same relationship hold when the lines are not parallel? Explain.

Alternate angles

7. Click the tab for page 3.



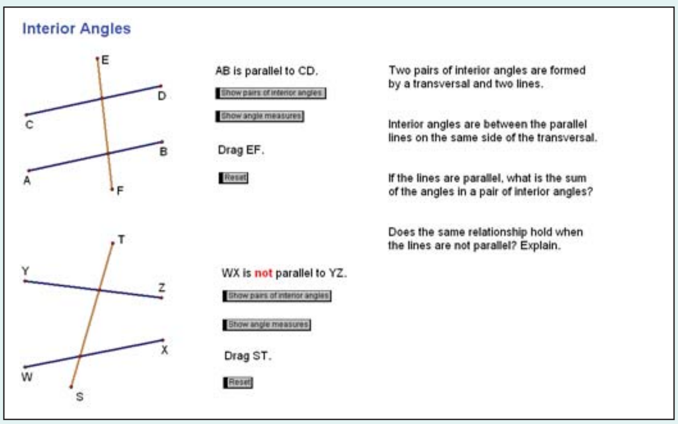
8. Two pairs of alternate angles are formed by a transversal and two lines.

a) If the lines are parallel, how are pairs of alternate angles related?

b) Does the same relationship hold when the lines are not parallel? Explain.

Interior angles

9. Click the tab for page 4.



10. Two pairs of interior angles are formed by a transversal and two lines.

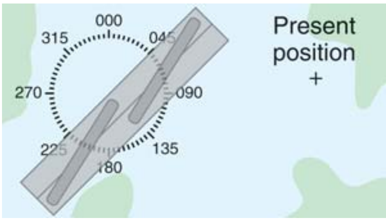
a) If the lines are parallel, what is the sum of the angles in a pair of interior angles?

b) Does the same relationship hold when the lines are not parallel? Explain.

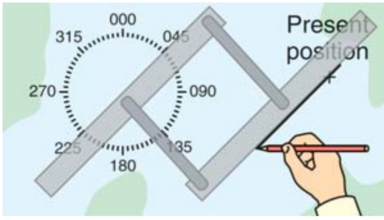
3.3 Angles Involving Parallel Lines

Jon is learning to sail. He uses parallel rulers to map a course 225° from his current position.

He places one edge of a ruler to align with 225° on a compass.



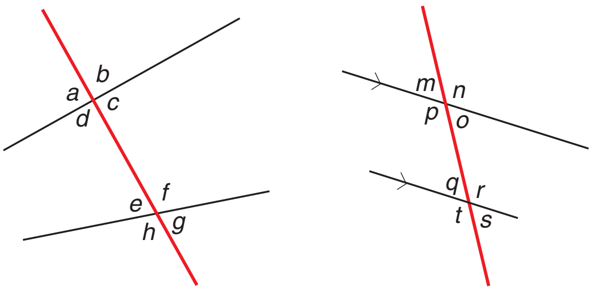
He swings the other edge of the ruler to his current position and draws the course line.



Investigate Angles Involving Parallel Lines

You will need a protractor.

➢ Draw diagrams like this.
Use both edges of a ruler to draw the parallel lines in the second diagram.



➢ Measure the angles.

➢ How are the angles related?

➢ Which angles have a sum of 180°? How do you know?

Reflect

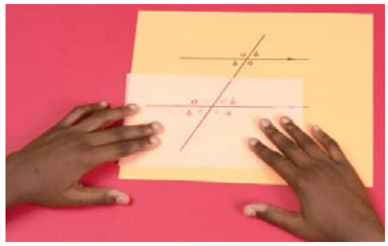
➢ When a line intersects two parallel lines, how can you remember which pairs of angles are equal?

➢ Does each diagram have the same number of pairs of equal angles? Explain.

Connect the Ideas

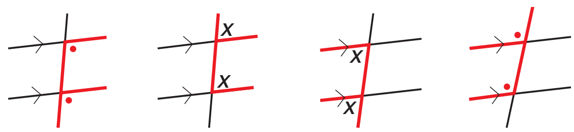
A transversal is a line that intersects two or more other lines.

When a **transversal** intersects two lines, four pairs of opposite angles are formed. The angles in each pair are equal. When the lines are parallel, angles in other pairs are also equal. We can use tracing paper to show these relationships.



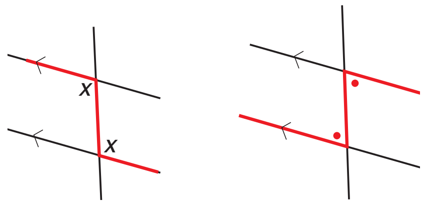
Corresponding angles form an F pattern.

Corresponding angles are equal. They have the same position with respect to the transversal and the parallel lines.



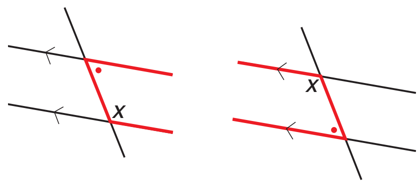
Alternate angles form a Z pattern.

Alternate angles are equal. They are between the parallel lines on opposite sides of the transversal.

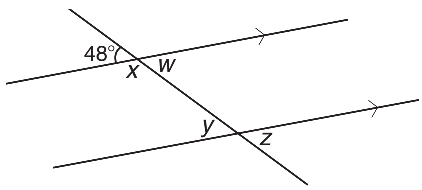


Interior angles form a C pattern.

Interior angles have a sum of 180°. They are between the parallel lines on the same side of the transversal.



We can use these relationships to determine the measures of other angles when one angle measure is known.

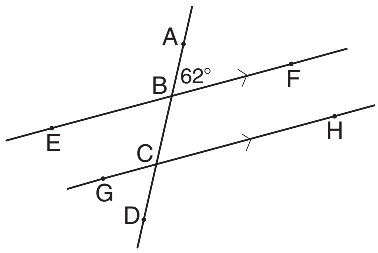


- w and 48° are measures of opposite angles. Opposite angles are equal, so $w = 48°$.
- w and x form a straight angle. The sum of their measures is 180°.
  $w = 48°$, so $x = 180° - 48°$
  $= 132°$
- w and y are alternate angles. They are equal.
  $w = 48°$, so $y = 48°$
- w and z are corresponding angles. They are equal.
  $w = 48°$, so $z = 48°$

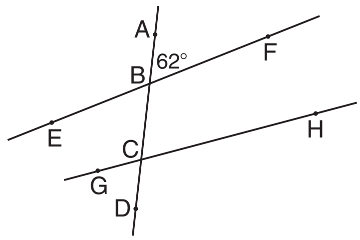
Practice

1. In each diagram, which other angle measures can you find? Determine these measures. Which relationships did you use?

a)



b)



Example Determine the values of *x* and *y*.

Solution Since *x* and 116° are the measures of interior angles between parallel lines, their sum is 180°; that is, $x + 116° = 180°$

So, $x = 180° - 116°$

$= 64°$

Similarly, the measures of *y* and 134° have a sum of 180°; that is,

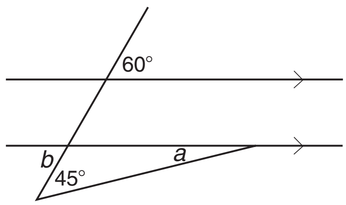
$y + 134° = 180°$

So, $y = 180° - 134°$

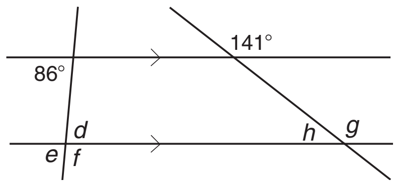
$= 46°$

2. Determine the angle measure indicated by each letter. Justify your answers.

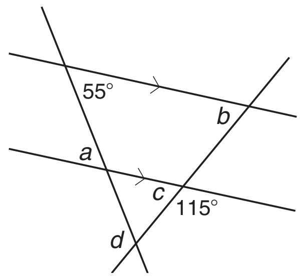
a)



b)



c)



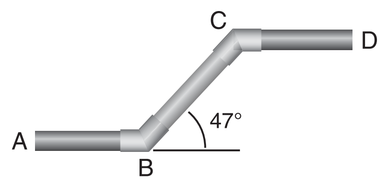
3. Assessment Focus Petra says these relationships are always true when two parallel lines are intersected by a transversal.

- All the acute angles are equal.
- All the obtuse angles are equal.
- The sum of any acute angle and any obtuse angle is 180°.

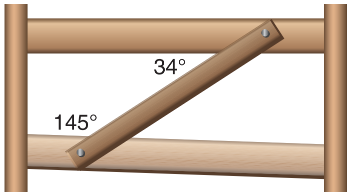
Is she correct?
Explain your thinking.
Include a diagram in your explanation.

Which relationships can you use?

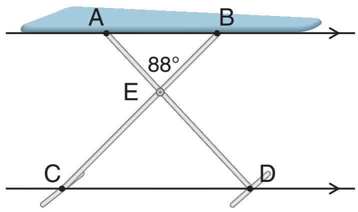
4. The diagram shows three sections of a steam pipe.
The top pipe and bottom pipe are parallel.
What is the angle between sections BC and CD?
Show your work.



5. Alison is building a section of fence.
To make sure the boards are parallel,
she measures the angles indicated.
Are the boards parallel?
Justify your answer.



6. An ironing board is designed so the board is always parallel to the floor as it is raised and lowered.
In the diagram, the two triangles are isosceles and ∠AEB is 88°.
Determine all the angles in △ECD.



7. a) List what you know about the interior and exterior angles of triangles.

b) List what you know about angles involving parallel lines cut by a transversal.

c) Use the properties in parts a and b.
Create a design, pattern, or picture.
Colour your design.
Explain how it illustrates the properties.

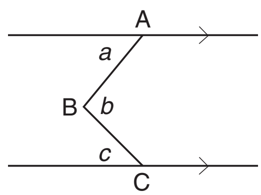
Which tools could you use to help you?

8. **Take It Further** Use a ruler to draw a diagram like the one at the right.
Two transversals intersect at B.

a) Measure to determine the values of a, b, and c.
How are the measures related?

b) Draw another diagram like this, so that a, b, and c have different values. Are a, b, and c related in the same way as in part a? Explain.

c) Explain why the relationship is always true.



In Your Own Words

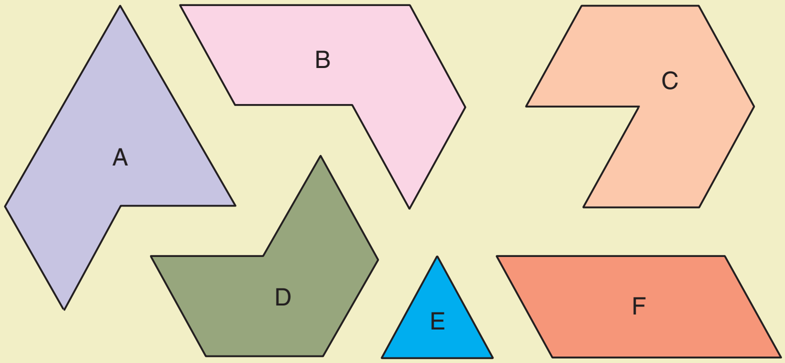
Research to find how geometry is used in real life.
You could look at occupations, sports, or architecture.
Use words, pictures, or numbers to explain what you found out.

Polygon Pieces

PUZZLE

Try this puzzle after you have completed Section 3.5.

The two puzzles on this page involve these 6 polygons.

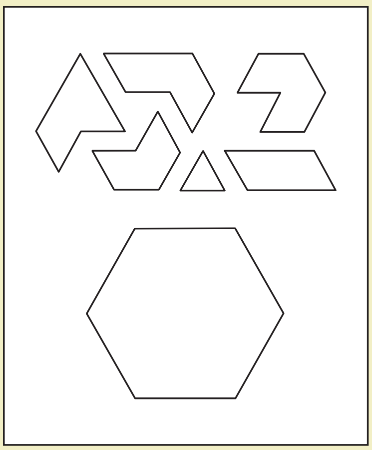


Puzzle 1

The sum of the interior angles is the same for two of the polygons.
Which polygons are they?
How can you tell without measuring any of the angles and without calculating the sum of the interior angles of any of the polygons?

Puzzle 2

Your teacher will give you a copy of the 6 polygons.
There is also a large regular hexagon on the copy.

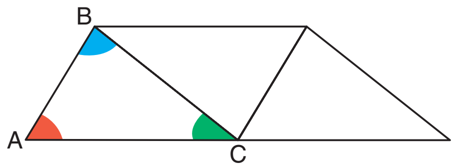


- Glue your copy to a piece of cardboard.
  Then cut out the 6 polygons.
- Arrange the 6 polygons to fit together on the regular hexagon.
  Record your solution to the puzzle on triangular dot paper.
- The puzzle can be solved in different ways.
  Compare your solution with those of other students.
  Record as many solutions as you can on triangular dot paper.

Mid-Chapter Review

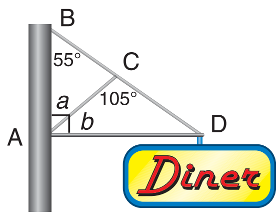
3.1

1. Your teacher will give you a copy of the diagram below.
The 3 triangles are congruent.



Colour red all angles equal to ∠A.
Colour blue all angles equal to ∠B.
Colour green all angles equal to ∠C.
Use the diagram to explain why the sum of the angles in a triangle is 180°.

2. Triangles ABC and ACD are braces for the "Diner" sign.
What are the values of *a* and *b*?

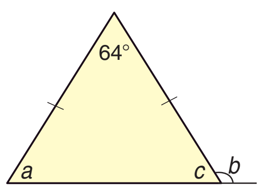


3.2

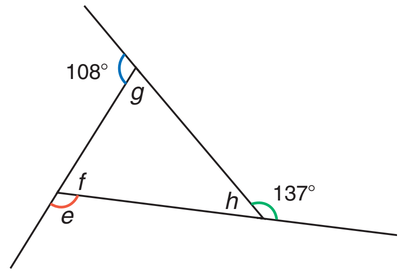
3. Use your diagram from question 1. Explain why the exterior angle of a triangle is equal to the sum of the two opposite interior angles.

4. Determine the angle measure indicated by each letter.

a)



b)



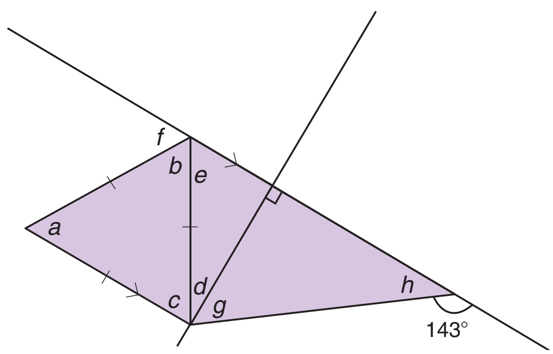
5. a) Draw a triangle and its exterior angles.
b) Measure and record enough angles so that someone could determine the remaining angles.
c) Trade diagrams with a classmate. Solve your classmate's problem.

3.3

6. Here is the flag of Antigua.

Determine the angle measure indicated by each letter.
What assumptions are you making?

7. a) Write all the relationships you can for the angles in this diagram.

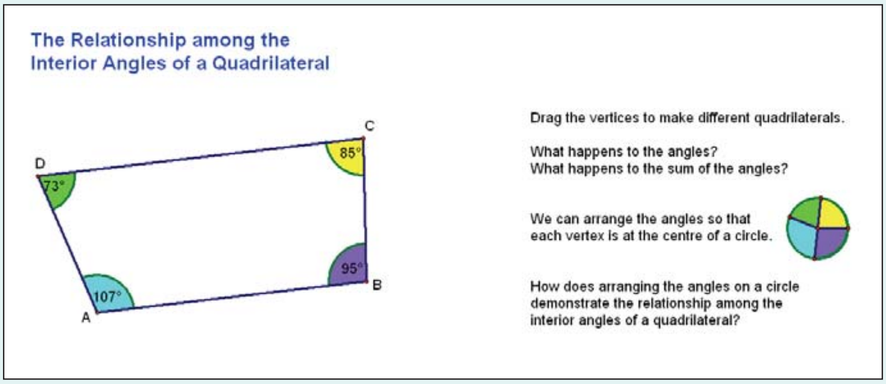


b) Use these relationships to determine the angle measure indicated by each letter.

Using *The Geometer's Sketchpad* to Investigate Angles Related to Quadrilaterals

Part 1: The Relationship among the Interior Angles

1. Open the file *QuadAngles.gsp*. Click the tab for page 1.



2. Drag vertices A, B, C, and D to create 5 different quadrilaterals.

a) Copy this table. For each quadrilateral, record the measures of ∠A, ∠B, ∠C, and ∠D.

∠A	∠B	∠C	∠D	∠A + ∠B + ∠C + ∠D

b) Calculate the sum of each set of angles.
Record each sum in the table.

c) How do the interior angles of a quadrilateral appear to be related? Explain.

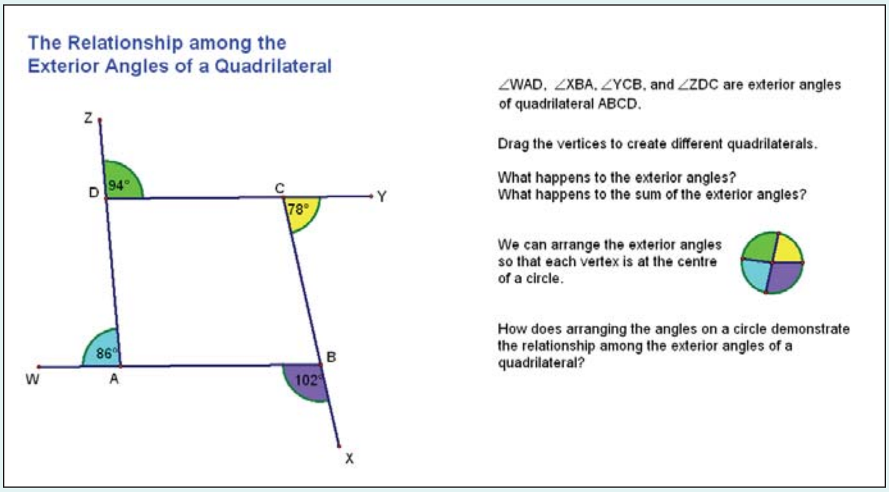
3. We can arrange ∠A, ∠B, ∠C, and ∠D so the vertex of each angle is at the centre of a circle.

a) Drag vertices A, B, C, and D to create different quadrilaterals. Observe how the angles change on the circle.

b) How does arranging the angles on a circle support your answer to question 2c?

Part 2: The Relationship among the Exterior Angles

1. Use the file *QuadAngles.gsp*. Click the tab for page 2.



2. Drag vertices A, B, C, and D to create 5 different quadrilaterals.

a) Copy this table. For each quadrilateral, record the measures of the exterior angles.

∠WAD	∠XBA	∠YCB	∠ZDC	∠WAD + ∠XBA + ∠YCB + ∠ZDC

b) Calculate the sum of each set of angles.
Record each sum in the table.

c) How do the exterior angles of a quadrilateral appear to be related? Explain.

3. We can arrange the exterior angles so each vertex is at the centre of a circle.

a) Drag vertices A, B, C, and D to create different quadrilaterals. Observe how the angles change on the circle.

b) How does arranging the angles on a circle support your answer to question 2c?

3.4 Interior and Exterior Angles of Quadrilaterals

Investigate Angle Relationships in Quadrilaterals

Which tools could you use to help you?

➢ Draw a quadrilateral.
Measure the interior angles.
Calculate their sum.

➢ Extend each side of the quadrilateral to draw 4 exterior angles, one at each vertex.
Measure the exterior angles.
Calculate their sum.

➢ Repeat the steps above for 2 different quadrilaterals.
What is the sum of the interior angles of a quadrilateral?
What is the sum of the exterior angles of a quadrilateral?

➢ What is the relationship between the interior angle and the exterior angle at any vertex of a quadrilateral?

Reflect

➢ Did your results depend on the type of quadrilateral drawn? Explain.

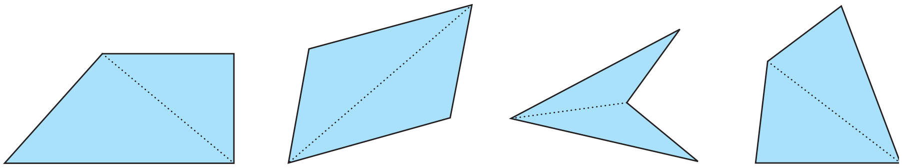
➢ How can you use what you know about the sum of the angles in a triangle to determine the sum of the angles in a quadrilateral?

➢ How can you use the sum of the interior angles of a quadrilateral to determine the sum of the exterior angles?

Connect the Ideas

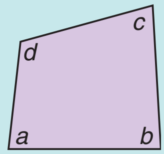
Interior angles of a quadrilateral

Any quadrilateral can be divided into 2 triangles.



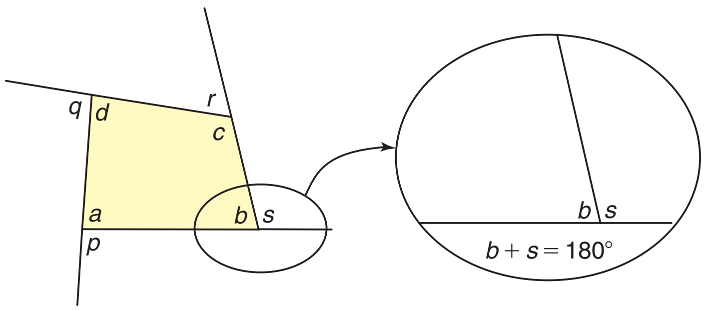
The sum of the angles in each triangle is 180°.
The sum of the angles in 2 triangles is: $2 \times 180° = 360°$

So, the sum of the interior angles of any quadrilateral is 360°.
That is, $a + b + c + d = 360°$



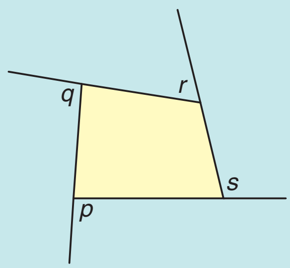
Exterior angles of a quadrilateral

At each vertex, an interior angle and an exterior angle form a straight angle.
The sum of their measures is 180°.



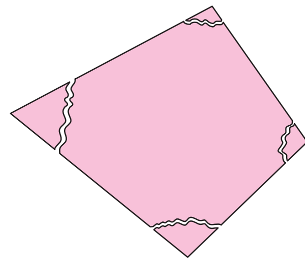
A quadrilateral has 4 vertices.
So, the sum of the interior and exterior angles is:
$4 \times 180° = 720°$
The sum of the interior angles is 360°.
So, the sum of the exterior angles is:
$720° - 360° = 360°$

The sum of the exterior angles is 360°.
That is, $p + q + r + s = 360°$

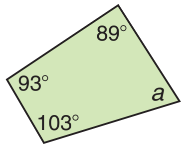


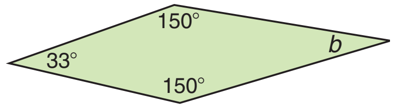
Practice

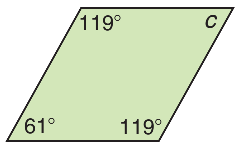
1. Draw then cut out a large quadrilateral.
Tear off the corners.
How can you arrange the corners to show that the interior angles of a quadrilateral have a sum of 360°?
Sketch what you did.



2. Determine the angle measure indicated by each letter.

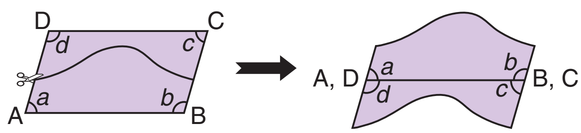
a) 

b) 

c) 

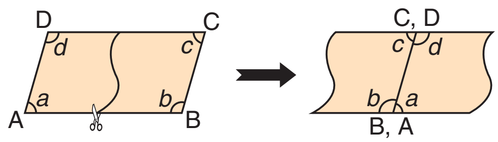
3. Draw 2 congruent parallelograms. Cut them out.

a) Cut out and arrange the pieces of one parallelogram as shown.



Which tools could you use to help you?

Cut out and arrange the pieces of the second parallelogram as shown.

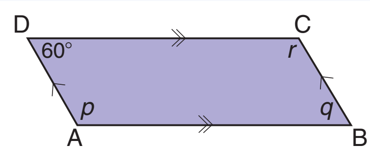


How are two angles that share a common side of a parallelogram related?

b) Use what you know about angles involving parallel lines to explain the relationship in part a.

Example

One angle of a parallelogram is 60°. Determine the measures of the other angles.



Solution

Since p and 60° are the measures of interior angles between parallel sides DC and AB, their sum is 180°; that is, $p + 60° = 180°$

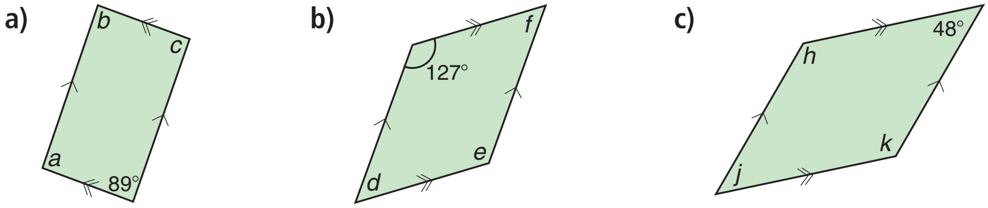
So, $p = 180° - 60°$
$= 120°$

Similarly, p and q are the measures of interior angles between parallel sides AD and BC.

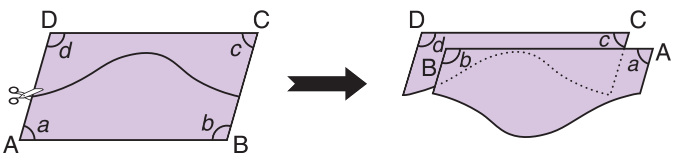
So, $q + 120° = 180°$, then $q = 60°$

q and *r* are the measures of interior angles between parallel sides CD and BA.
So, $60° + r = 180°$, then $r = 120°$

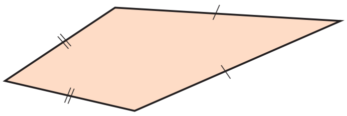
4. Determine the angle measure indicated by each letter.
Which relationships did you use each time?



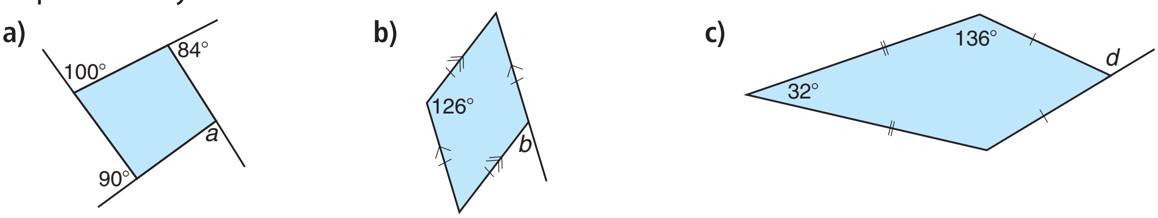
5. Draw then cut out a parallelogram.
Cut out and arrange the pieces of the parallelogram as shown.
What does this tell you about opposite angles in a parallelogram?



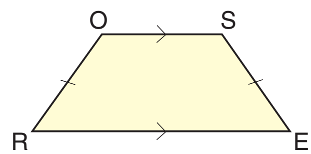
6. A kite has pairs of adjacent sides equal.
a) Draw a kite. Determine how the angles are related.
Explain how you did this.
b) Compare your answer to part a with that of a classmate.
Work together to explain why the relationships in part a are always true.



7. **Assessment Focus** Determine the angle measure indicated by each letter.
Explain how you know.



8. **Take It Further** Quadrilateral ROSE is an isosceles trapezoid.
Sides OS and RE are parallel. Sides OR and SE are equal.
a) Find as many relationships as you can among the interior angles of ROSE.
b) Choose a tool and use it to demonstrate the relationships in part a.



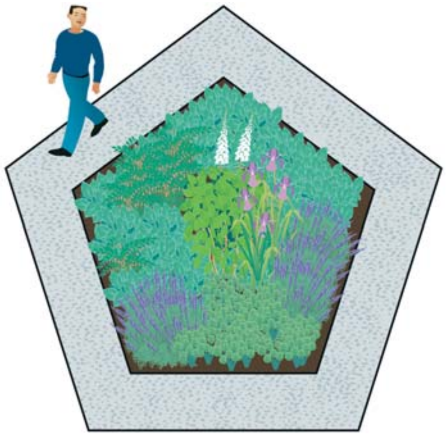
In Your Own Words

What do you know about the interior and exterior angles of quadrilaterals? Use diagrams to explain.

3.5 Interior and Exterior Angles of Polygons

A garden has the shape of a pentagon.
It is surrounded by straight walking paths.
James starts walking around the garden.

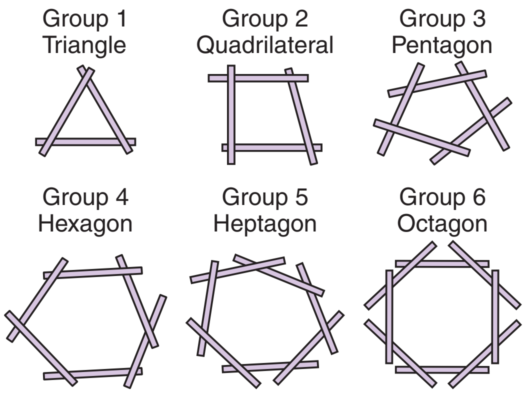
At each fork, he turns left.
When James returns to his starting point,
through how many degrees has he turned?



Investigate — The Sum of the Exterior Angles of a Polygon

Which tools could you use to help you?

➢ Work in a group.
You will need a large open space.
Your group will be assigned one of the polygons below.
Use masking tape to create it.



➢ Have a member of the group walk around the polygon until she arrives back where she started from, and faces the way she began. Determine the total angle she turned through.

➢ Repeat for each of the other polygons.
What do you notice?

Reflect

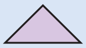
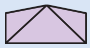
➢ Do the results depend on the number of sides in the polygon? Explain.

➢ What do these results suggest about the sum of the exterior angles of any polygon? Explain.

Connect the Ideas

Interior angles of a polygon

We can determine the sum of the interior angles of any polygon by dividing it into triangles.

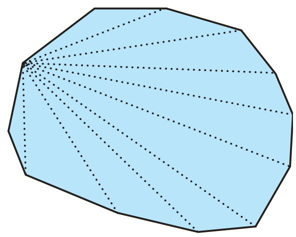
Polygon	Number of sides	Number of triangles	Angle sum
Triangle 	3	1	1(180°) = 180°
Quadrilateral 	4	2	2(180°) = 360°
Pentagon 	5	3	3(180°) = 540°
Hexagon 	6	4	4(180°) = 720°

In each polygon, the number of triangles is always 2 less than the number of sides. So, the angle sum is 180° multiplied by 2 less than the number of sides.
We can write this as an equation.

$S = (n - 2) \times 180°$
S is the sum of the interior angles of a polygon with n sides.

A polygon with 12 sides can be divided into 10 triangles.

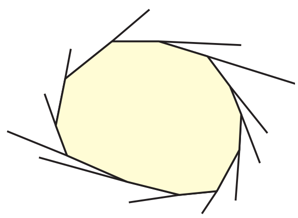
We can use this formula to determine the sum of the interior angles of a polygon with 12 sides.



$S = (n - 2) \times 180°$
Substitute: $n = 12$
$S = (12 - 2) \times 180°$
$= 10 \times 180°$
$= 1800°$
The sum of the interior angles of a 12-sided polygon is 1800°.

Exterior angles of a polygon

At each vertex, an interior angle and an exterior angle have a sum of 180°.
For 12 vertices, the sum of the interior and exterior angles is: $12 \times 180° = 2160°$
The sum of the interior angles is 1800°.



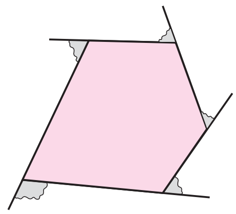
So, the sum of the exterior angles is: $2160° - 1800° = 360°$

We can reason this way for any polygon.

The sum of the exterior angles of any polygon is 360°.

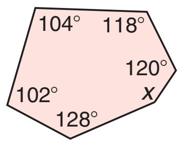
Practice

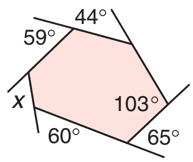
1. Use a ruler to draw a large pentagon and its exterior angles as shown. Cut out the exterior angles.
How can you arrange them to show that the exterior angles of a pentagon have a sum of 360°?
Sketch what you did.



2. What is the sum of the interior angles of a polygon with each number of sides?
   a) 7 sides b) 10 sides c) 24 sides

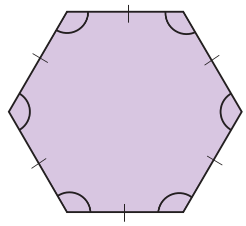
3. Determine the angle measure indicated by x.

a) 

b) 

Example
A regular polygon has all sides equal and all angles equal.
What is the measure of each interior angle in a regular hexagon?

Solution
A hexagon has 6 sides.
Use the sum of the interior angles:
$S = (n - 2) \times 180°$
Substitute: $n = 6$
$S = (6 - 2) \times 180°$
$= 4 \times 180°$
$= 720°$
Since the angles are equal, divide by 6 to determine the measure of each angle: $\frac{720°}{6} = 120°$
Each interior angle in a regular hexagon is 120°.



4. Determine the measure of one interior angle in a regular polygon with each number of sides.
   a) 5 sides b) 8 sides c) 10 sides d) 20 sides

5. Determine the measure of one exterior angle for each regular polygon in question 4.

6. A Canadian $1 coin has the shape of a regular polygon.
   a) How many sides does it have?
   b) What is the measure of each interior angle?
   Show your work.

7. **Assessment Focus**

a) List all that you know about the interior angles and exterior angles of polygons.

b) Use the properties in part a.
Create a design, pattern, or picture.
Colour your design to illustrate the properties.
Explain your colour scheme.

Use *The Geometer's Sketchpad* if available.

8. A concave polygon has an interior angle that is a reflex angle.
Here are two examples.

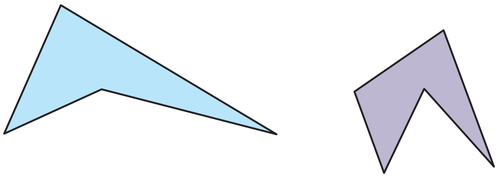
a) Draw a concave quadrilateral.

b) Draw a concave pentagon.

c) Measure the angles.
Determine the sum of the angles in each figure.

d) Do concave polygons obey this relationship:
Interior angle sum $= (n - 2) \times 180°$?
Justify your answer.

A reflex angle is between 180° and 360°.



9. Find a soccer ball or use this picture.

a) Which different polygons do you see on the ball?

b) What is the measure of one interior angle of a white polygon?

c) What is measure of one interior angle of a black polygon?

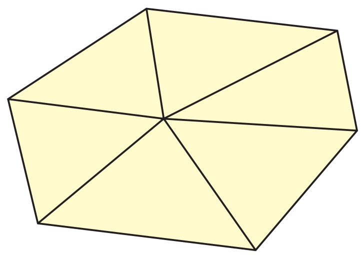
d) What is the sum of the angles at the red dot?
What do you notice?

10. **Take It Further** You can divide a hexagon into triangles by choosing a point inside the hexagon and joining all vertices to that point.

a) What is the sum of the angles in all the triangles?

b) What would have to be subtracted from the sum in part a to get the sum of the interior angles of the hexagon? Explain.

c) Repeat parts a and b for other polygons.
Write an equation for the sum of the interior angles in a polygon with n sides.



In Your Own Words

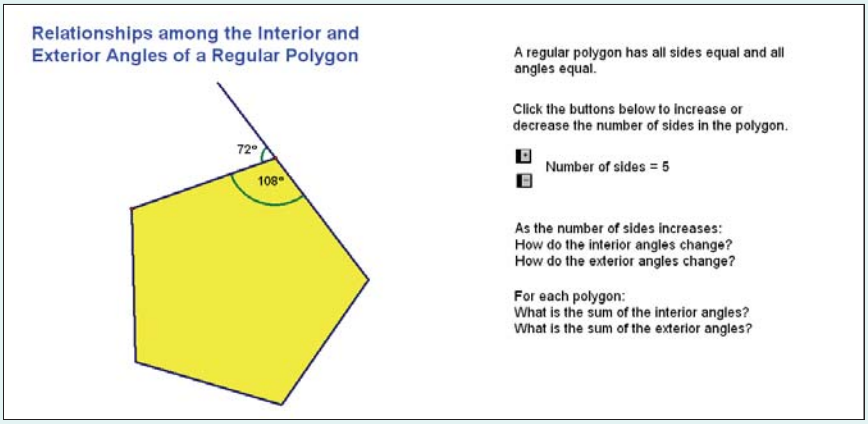
How can you find the sum of the angles in any polygon?
When can you use this sum to determine the measures of the angles in the polygon? Include sketches in your answer.

Using *The Geometer's Sketchpad* to Investigate Angles Related to Regular Polygons

Technology

Part 1: Relationships among the Interior Angles and the Exterior Angles of a Regular Polygon

1. Open the file *PolyAngles.gsp*. Click the tab for page 1.



2. A regular polygon has all sides equal and all angles equal.

a) Start with a regular polygon with 3 sides.
This is an equilateral triangle.
Copy the table.
Complete the first 3 columns for an equilateral triangle.

Number of sides in the regular polygon	Measure of one interior angle	Measure of one exterior angle	Sum of interior angles	Sum of exterior angles

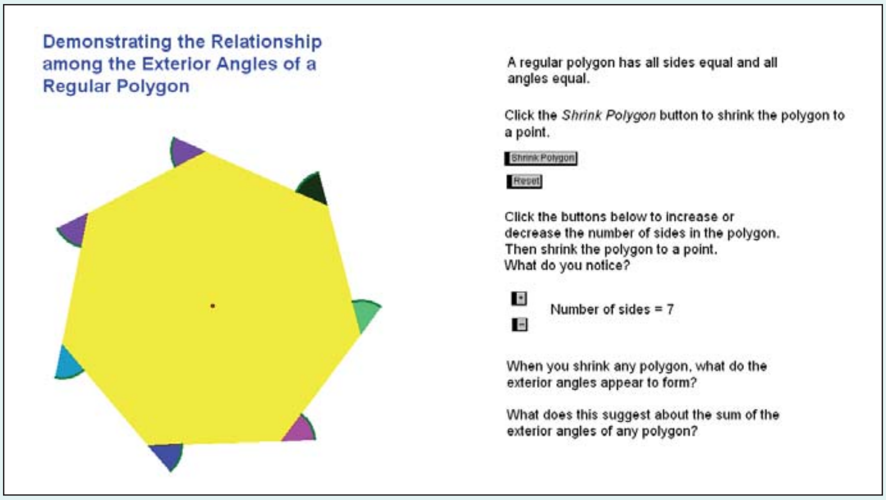
b) Use the + button to create a regular polygon with 4 sides.
Complete the first 3 columns of the table for this polygon.

c) Repeat part b for 4 different regular polygons.
As the number of sides increases:
How do the interior angles change?
How do the exterior angles change?

3. Complete the last two columns of the table for each polygon from question 2.
As the number of sides increases:
How does the sum of the interior angles change?
What do you notice about the sum of the exterior angles?

Part 2: The Relationship among the Exterior Angles of a Regular Polygon

1. Use the file *PolyAngles.gsp*. Click the tab for page 2.



2. In Part 1, question 3, you discovered a relationship among the exterior angles of a regular polygon.

a) Start with a triangle.
Click the *Shrink Polygon* button to shrink the triangle to a point.
What do the exterior angles appear to form?
Click the *Reset* button to return the polygon to its original size.

b) Click the + button to create a polygon with 4 sides.
Click the *Shrink Polygon* button to shrink the polygon to a point.
What do the exterior angles appear to form?
Click the *Reset* button to return the polygon to its original size.

c) Repeat the steps in part b for 4 different regular polygons.

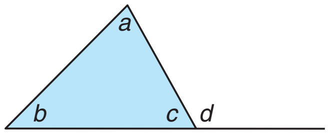
3. a) When you shrink any regular polygon, what do the exterior angles appear to form?

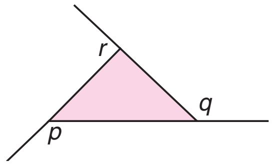
b) What does this suggest about the sum of the exterior angles of a regular polygon?

Chapter Review

What Do I Need to Know?

Any Triangle

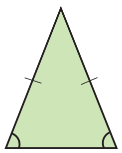




Sum of the interior angles is 180°.
$a + b + c = 180°$
Each exterior angle is the sum of the two opposite interior angles.
$d = a + b$

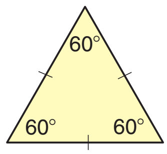
Sum of the exterior angles is 360°.
$p + q + r = 360°$

Isosceles Triangle



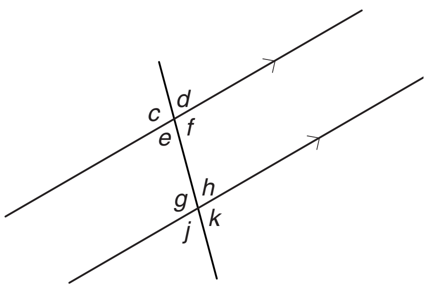
The angles opposite the equal sides are equal.

Equilateral Triangle



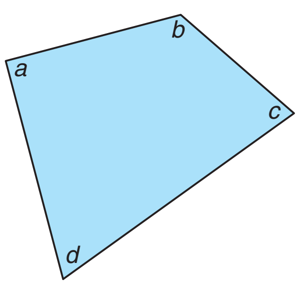
Each angle is 60°.

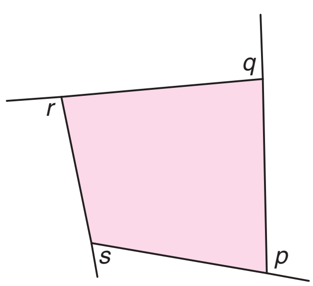
Parallel Lines



Corresponding angles are equal.
$c = g \quad e = j \quad d = h \quad f = k$
Alternate angles are equal.
$e = h \quad f = g$
Interior angles have a sum of 180°.
$e + g = 180° \quad f + h = 180°$

Any Quadrilateral

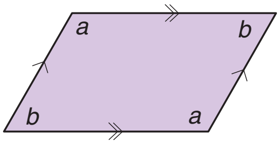




Sum of the interior angles is 360°.
$a + b + c + d = 360°$

Sum of the exterior angles is 360°.
$p + q + r + s = 360°$

Parallelogram



Opposite angles are equal.
Also, $a + b = 180°$

Any Polygon

If there are n sides, the sum of the interior angles is: $(n - 2) \times 180°$
The sum of the exterior angles is 360°, regardless of the number of sides.

What Should I Be Able to Do?

3.1

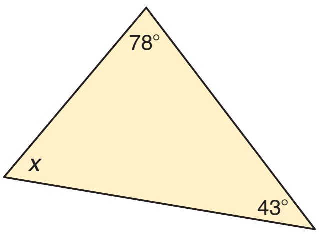
1. Explain how you know that the sum of the angles in a triangle is 180°.

2. Can a triangle have 2 obtuse angles? Explain.

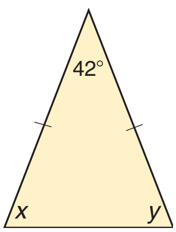
3.1 3.2

3. Determine the angle measure indicated by each letter.

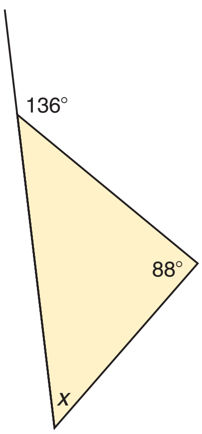
a)



b)

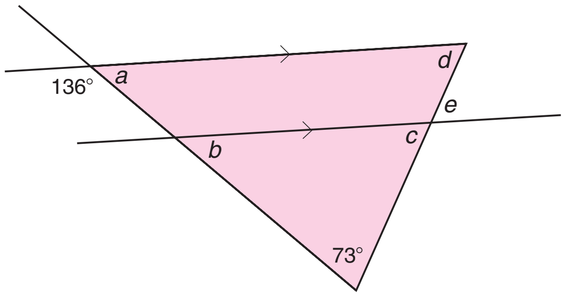


c)



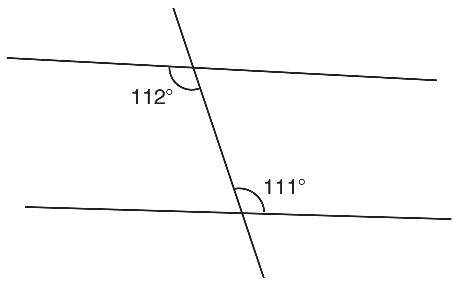
3.1 3.3

4. Determine the angle measure indicated by each letter.

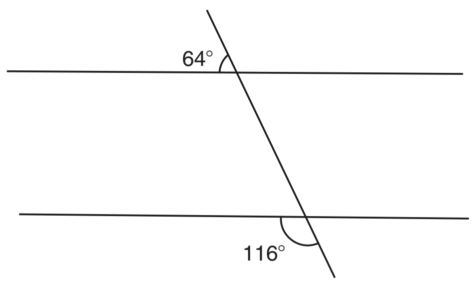


5. Are these lines parallel? Justify your answer.

a)



b)

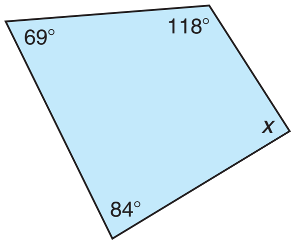


6. The top of this TV table is parallel to the ground. The triangles are isosceles. Determine the measures of the labelled angles.

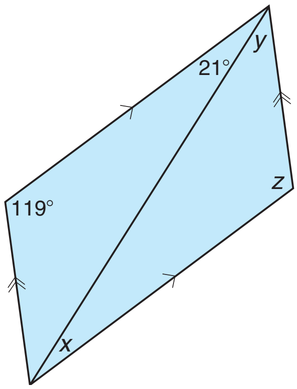
3.4

7. Determine the angle measure indicated by each letter. Justify your answer.

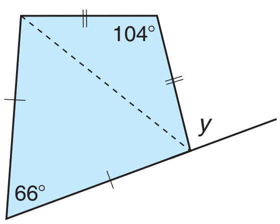
a)



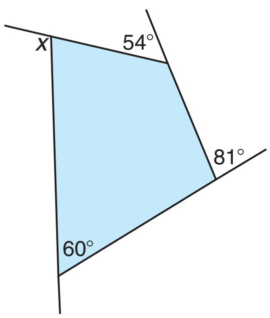
b)



c)

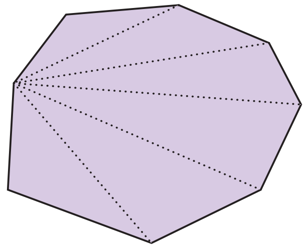


d)



3.5

8. a) Use this diagram to determine the sum of the interior angles in an octagon.



b) A regular octagon has 8 equal sides and 8 equal angles. Determine each measure:

i) an interior angle

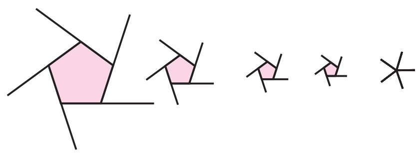
ii) an exterior angle

9. Determine the sum of the interior angles of a polygon with each number of sides. Try to do this two different ways.

a) 6 sides b) 12 sides c) 18 sides

10. a) What is the sum of the interior angles of a pentagon? Show your work.

b) What does the following sequence of pictures demonstrate about the sum of the exterior angles of a pentagon?



c) Does the result of part a agree with your answer in part b? Explain.

11. A regular polygon has 100 sides. Determine the measure of one interior angle and one exterior angle.

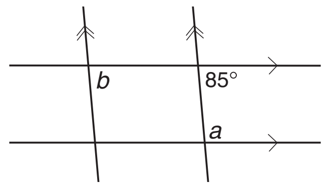
Practice Test

Multiple Choice: Choose the correct answer for questions 1 and 2.

1. What are the values of *a* and *b*?

A. $a = 85°, b = 85°$ **B.** $a = 95°, b = 85°$

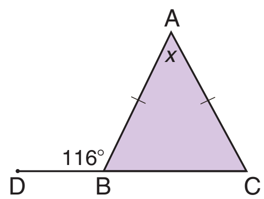
C. $a = 85°, b = 95°$ **D.** $a = 95°, b = 95°$



2. Determine the value of *x*.

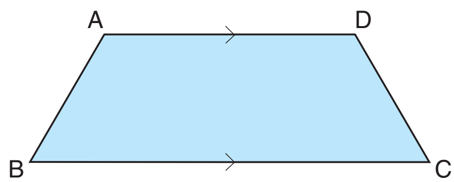
A. 32° **B.** 52°

C. 58° **D.** 64°



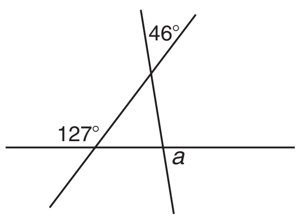
Show all your work for questions 3 to 6.

3. **Communication** Suppose you had a cutout of this quadrilateral. Explain how to demonstrate that the sum of its interior angles is 360°. How many different ways could you do this? Explain.

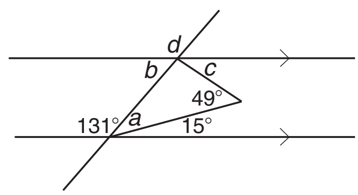


4. **Knowledge and Understanding** Determine the angle measure indicated by each letter.

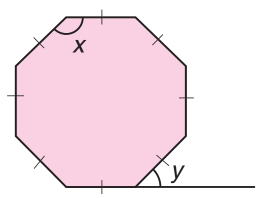
a)



b)



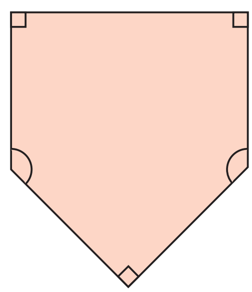
c)



5. **Application** On a baseball diamond, home plate has the shape of a pentagon. The pentagon has 3 right angles. The other 2 other angles are equal.

a) What is the sum of the angles of a pentagon?

b) What is the measure of each equal angle? How do you know?



6. **Thinking** Use this diagram. Explain why ∠PRQ = ∠RST.