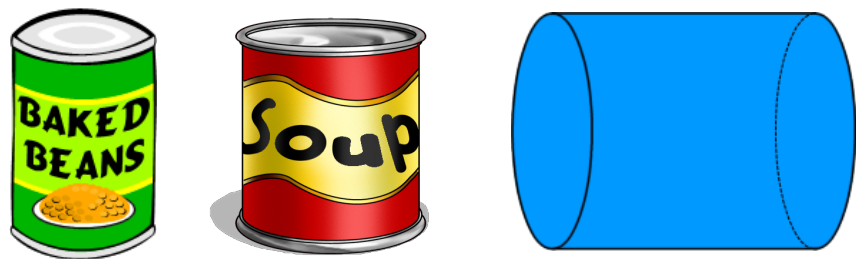


Investigation A :

How can you compare the volumes of cylinders with the same surface area?



Many products come in cylinders. Your task is to design a cylindrical juice can that uses no more than 375 cm² of aluminum. The can should have the greatest capacity possible.

Let's investigate...

1. Complete the table below.

NOTE: To investigate the volume of the cylinder as its radius changes, it will help to have an expression for the height in terms of the radius given that the surface area is 375 cm².

| Radius (cm) | Height (cm) | Volume (cm ³) | Surface Area (cm ²) |
|-------------|-------------|---------------------------|---------------------------------|
| 1 | | | 375 |
| 2 | | | 375 |
| 3 | | | 375 |
| 4 | | | 375 |
| 5 | | | 375 |
| 6 | | | 375 |
| 7 | | | 375 |

2. a) What is the maximum volume for the cans in your table?

b) What are the radius and height of the can with this volume?

REFLECT: Summarize your findings.

a) Do any relationships exist between the radius and height of a cylinder with maximum volume for a given surface area?

b) Do the dimensions found in the investigation give the optimal volume for the surface area of 375 cm²?

c) Can we determine the dimensions of a can with a volume greater than the value in the table?

d) How can you predict the dimensions of a cylinder with maximum volume if you know the surface area? Can we come up with a general formula?

EX. 1. Maximize the Volume of a Cylinder

a) Determine the dimensions of the cylinder with maximum volume that can be made with 600 cm² of aluminum. Round the dimensions to the nearest hundredth of a centimetre.

b) What is the volume of this cylinder to the nearest cubic centimetre?

Investigation B :

How can you compare the surface areas of cylinders with the same volume?

Your task is to design a cylindrical juice can that must hold 1000 mL of juice.

1. Complete the table below.

NOTE: To investigate the surface area of the cylinder as its radius changes, it will help to have an expression for the height in terms of the radius given that the volume is 1000 mL.

| Radius (cm) | Height (cm) | Volume (1 cm ³ = 1 mL) | Surface Area (cm ²) |
|-------------|-------------|--------------------------------------|------------------------------------|
| 1 | | 1000 | |
| 2 | | 1000 | |
| 3 | | 1000 | |
| 4 | | 1000 | |
| 5 | | 1000 | |
| 6 | | 1000 | |
| 7 | | 1000 | |

2. a) What is the minimum surface area for the cans in your table?

b) What are the radius and height of the can with this surface area?

REFLECT: Summarize your findings.

- a) Do any relationships exist between the radius and height of a cylinder with minimum surface area for a given volume?
- b) Do the dimensions found in the investigation give the optimal surface area for the volume of 1000 mL?
- c) Can we determine the dimensions of a can with a surface area smaller than the value in the table?
- d) How can you predict the dimensions of a cylinder with minimum surface area if you know the volume? Can we come up with a general formula?

EX. 2. Minimize the Surface Area of a Cylinder

- a) Determine the least amount of aluminum required to construct a cylindrical container with a 4L capacity, to the nearest tenth of a square centimetre.

RECALL: $1 \text{ mL} = 1 \text{ cm}^3$ and because $1 \text{ L} = 1000 \text{ mL}$, therefore $1 \text{ L} = 1000 \text{ cm}^3$

- b) Describe any assumptions you made.

Practice: p. 508 #1-4 & p. 513 #1, 2, 5, 6