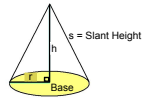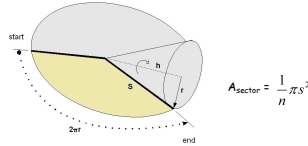A <u>cone</u> is a three dimensional solid with a circular base. The lateral surface is curved and extends from the base to a point called the vertex.


s = Slant Height

Developing a formula for surface area of a cone:

The lateral surface is a circle-sector.
This sector is some fraction (one n^{th}) of a circle with radius s.

$$A_{sector} = \frac{1}{n}\pi s^2$$

The circumference of the sector is one n^{th} of the circumference the whole circle with radius s.

$$C_{sector} = \frac{1}{n}(2\pi s)$$

Since the circumference of the sector wraps around the circumference of the base (which is a circle with radius r),
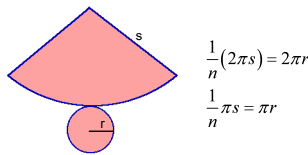
$$\frac{1}{n}(2\pi s) = 2\pi r$$

$$\frac{1}{n}\pi s = \pi r$$

substituting this into $A_{sector} = \frac{1}{n}\pi s^2$

$$A_{lateral\ side} = \frac{1}{n}\pi s \times s$$
$$= \pi r \times s$$
$$= \pi rs$$

So, the formula for Surface area of a cone is:

$$A_{total} = A_{base} + A_{lateral\ side}$$
$$= \pi r^2 + \pi rs$$

<u>Example 1:</u>

Calculate the surface area of a paper cone (before it is filled with french fries) with height 4.2 cm and radius 1.8 cm.

** Note: the paper cone has no circular base so we only need the lateral surface area!!

$$SA = \cancel{\pi r^2} + \pi r s$$

$$a^2 + b^2 = c^2$$
$$1.8^2 + 4.2^2 = c^2$$
$$3.24 + 17.64 = c^2$$
$$\sqrt{20.88} = c^2$$
$$4.57 = c$$

$$SA = \pi r s$$
$$= 3.14(1.8)(4.57)$$
$$= 25.82\ cm^2$$

<u>Example 2:</u>

The slant height of a cone is tripled. Does this triple the surface area of the cone? Explain.
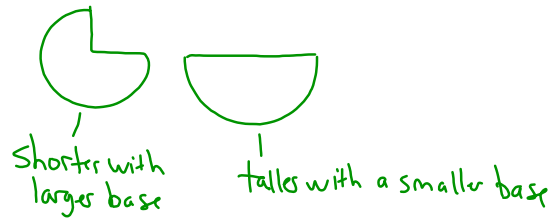
$$SA = \pi r^2 + \pi r s$$
$$\pi r^2 + \pi r s(3)$$

No, it only triples the lateral side

Example 3:
A cone is formed from a circle with a 90° sector removed. Another cone is formed from a semicircle with the same radius. How do the two cones differ? How are they the same?

−Slant height will be the same

Shorter with larger base

taller with a smaller base

Example 4: The lateral area of a cone with slant height 14 cm is 132 cm².

a) Find the radius of the cone, to the nearest cm.

b) Find the height of the cone, to the nearest cm.

Example 5:
An old construction pylon needs to be painted. The base the pylon sits on is 20cm by 20 cm by 1.5 cm, the radius of the cone is 8 cm and the height of the pylon is 31 cm. If only the part that shows is to be painted, find the surface area to be painted. (Round to 1 decimal place).
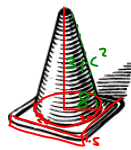
SA = square base − $\frac{cone}{base}$ + cone

SA = square base
$= (20 \times 20) + (1.5 \times 20 \times 4)$
$= 400 + 120$
$= 520 \text{ cm}^2$

SA = cone base
$= \pi r^2$
$= 3.14(8^2)$
$= 200.96 \text{ cm}^2$

SA = lateral surface of cone
$= \pi r s$
$= 3.14(8)(32.02)$
$= 804.3 \text{ cm}^2$

$8^2 + 31^2 = c^2$
$64 + 961 = c^2$
$1025 = c^2$
$32.02 = c$

SA = 520 − 200.96 + 804.34
SA = 1123.38 cm²
SA = 1123.4 cm²