A <u>cylinder</u> is

a three dimensional solid with identical parallel circular bases. The lateral surface is curved and extends from one base to the other base.

The volume of a cylinder is the same as a prism:

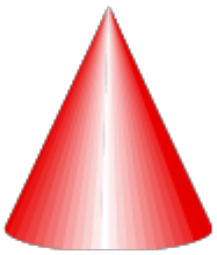
$V = A_{base} \times \text{height}$

or $V = \pi r^2 h$

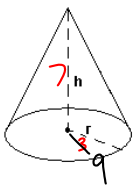
Similar to the relationship between the pyramid and the prism, the volume of a cone is one third the volume of a cylinder with the same radius and height.

Volume of a cone = $A_{base} \times \text{height}$

$= \frac{\pi r^2 h}{3}$ or $\frac{1}{3}\pi r^2 h$



<u>Example 1</u>: Calculate the volume of a cone with radius 3 mm and height 7 mm.



$V = \frac{\pi r^2 h}{3}$

$V = \frac{3.14(3^2)(7)}{3}$

$V = \frac{3.14(9)(7)}{3}$

$V = 65.94\ mm^3$

<u>Example 2:</u>

a) If the height of a cone is tripled, does this triple the volume? Explain.

$V = \frac{3.14(3^2)(21)}{3}$

$= 197.82\ mm^3$

$64.94 \times 3 = 197.82$

Yes because we are multiplying the other thing by 3

b) If the radius of a cone is tripled, does this triple the volume? Explain.

$V = \frac{3.14(9^2)(7)}{3}$

$V = 593.46\ mm^3$

No because we changed the number with the exponent which changes the answer drastically

Example 3: A grain bin has a radius of 12 ft and a height of 48 ft. How much grain will the farmer need to order to fill the bin? (Note: 1 kg of feed fills 1 ~~ft³~~ of space. Also, assume grain (oats) is ordered in tonnes (1 metric ton = 1000kg) .)
(Note: the height of the cone portion is 18 ft.)

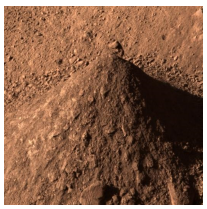
Cylinder
$V = \pi r^2 h$
$V = 3.14(12^2)(30)$
$V = 3.14(144)(30)$
$V = 13,564.8\ ft^3$

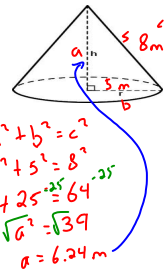
Cone
$V = \frac{\pi r^2 h}{3}$
$V = \frac{3.14(12)^2(18)}{3}$
$V = 2712.96\ ft^3$

Total volume = V of cylinder + V of cone
$= 13,564.8 + 2712.96$
$= 16,277.76\ ft^3$
$= 16,277.76\ kg$
$= 16.3$ tons

∴ 17 tons would be needed to fill the grain bin

Example 4: A conical pile of sand has a base diameter of 10 m and a slant height of 8 m. Determine the volume of the sand in the pile, to the nearest cubic metre.



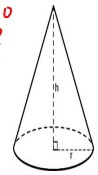


$a^2 + b^2 = c^2$
$a^2 + 5^2 = 8^2$
$a^2 + 25 = 64$
$\sqrt{a^2} = \sqrt{39}$
$a = 6.24\ m$

$r = 5$
$h = 6.24$
$V = \frac{\pi r^2 h}{3}$
$V = \frac{3.14(5^2)(6.24)}{3}$
$V = 163.28\ m^3$

Example 5: A fountain firework is packaged in a conical container. Its volume is 210 cm³. Its diameter is 8 cm. What is the height of the fountain firework, to the nearest tenth of a centimeter?

$V = \frac{\pi r^2 h}{3}$
$V = 210$
$D = 8$
$r = 4$



$\times 3$
$210 = \frac{3.14(4^2)(h)}{3}$ $\times 3$
$630 = 3.14(4^2)(h)$
$\frac{630}{16} = \frac{3.14(16)(h)}{16}$
$\frac{39.375}{3.14} = \frac{3.14(h)}{3.14}$
$12.54 = h$

∴ the height would be 12.5 cm